Let me recap the rules we discussed in the last post:

Rule 1: $a^m \times a^n = a^{(m + n)}$

Rule 2: $a^m / a^n = a^{(m - n)}$

Rule 3: $(a^m)^n = a^{mn}$

Rule 4: For any number a, $a^0 = 1$

In Rules 1 and 2 above, the bases were the same. What happens if the exponents are the same but the bases are different?

2^4 × 3^4 = (2 × 2 × 2 × 2) × (3 × 3 × 3 × 3) = (2 × 3) × (2 × 3) × (2 × 3) × (2 × 3) = 6^4

The bases got multiplied!

Rule 5: a^m × b^m = (a × b)^m

which also implies that (a × b)^m = a^m × b^m

Example:

6^5 = 2^5 * 3^5

The treatment of division is very similar.

6^4/2^4 = (6 × 6 × 6 × 6)/(2 × 2 × 2 × 2) = 3 × 3 × 3 ×3 = 3^4

Essentially, the bases get divided and the exponent remains the same.

Rule 6: a^m / b^m = (a / b)^m

which also implies that (a / b)^m = a^m / b^m

Example:

(3/2)^5 = 3^5 / 2^5

Let's take a quick look at negative exponents now.

What is 3^{-4}? It is extremely easy to handle negative exponents. Just flip the number and the exponent becomes positive. So 3^{-4} is $1/3^4$. This also implies the following:

- $1/3^{-4} = 3^4$
- $3^4 = 1/3^{-4}$
- $1/3^4 = 3^{-4}$

When you want to change the sign of the exponent, flip the number. When you want to flip the number, change the sign of the exponent!

Example: What is the value of 3^2 / 3^(-3)

This can be solved in two ways:

1. Flip the number to get rid of the negative exponent.

3^2 / 3^(-3) = 3^2 * 3^3

The base is the same and the two are multiplied so we add the exponents.

3^2 * 3^3 = 3^(2+3) = 3^5

OR

2. The bases are the same and the terms are divided so subtract the exponent of the divisor from the exponent of the dividend.

3^2 / 3^(-3) = 3^(2 – (-3)) = 3^5

Now that we have covered all the major rules of exponents, let's work on the question we saw in the previous post.

Question: If $(2^a \times 4 \times 3^{-4} \times 3^b)/(3^4 \times 2^2) = 8^{-4} \times 729$, what is the value of a and b?

(A) -10 and -14
(B) 10 and 12
(C) -10 and 12
(D) -12 and 14
(E) 12 and -14

Solution: First, we need to bring everything to prime number form.

$(2^a \times 4 \times 3^{-4} \times 3^b)/(3^4 \times 2^2) = 8^{-4} \times 729$

$(2^a \times 2^2 \times 3^{-4} \times 3^b)/(3^4 \times 2^2) = (2^3)^{-4} \times 3^6$

If you do not remember that 729 is the sixth power of 3, you should know that it will be some power of 3 because the left hand side of the equation has only two prime numbers – 2 and 3. So the right hand side of the equation cannot have any prime other than 2 and 3 (if there is some other prime number, its exponent will be 0 to make the term 1). Since 729 is certainly not a power of 2 (since it is odd), it must be some power of 3. We just need to multiply 3 with itself a few times to figure out the power.

Let's work on the left hand side of the equation first. Get rid of negative exponent.

$(2^a \times 2^2 \times 3^b)/(3^4 \times 3^4 \times 2^2)$

Some terms have same bases and are multiplied. Add their exponents.

(2^(a + 2) × 3^b)/(3^8 × 2^2)

Some terms have same bases and are divided. Subtract their exponents.

2^a × 3^(b – 8)

Let’s equate this to the right hand side now.

$2^a \times 3^{(b-8)} = (2^3)^{-4} \times 3^6$

Using Rule 3 on the right hand side,

$2^a \times 3^{(b-8)} = 2^{(-12)} \times 3^6$

The exponent of 2 on left hand side is ‘a’ and on right hand side is -12. Therefore, a = -12.

The exponent of 3 on left hand side is (b – 8) and on right hand side is 6. Therefore, b – 8 = 6 or b = 14

Answer (D)

Note: We used the long method to solve this question since we wanted to discuss the application of various rules. You can use faster approaches once you are comfortable with these basic rules.

Let me leave you with a couple of questions. I will discuss these in the next post.

Question 1: Given $(1/4)^{18} \times (1/5)^n = 1/(2 \times 10^{35})$, find the value of n.

Question 2: Is $5^m < 1000$?

Statement 1: $5^{(m+1)} > 3000$

Statement 2: $5^{(m-1)} = 5^m - 500$